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GROWTH LAW OF A SPHERICAL SECOND PHASE AS GOVERNED BY SIMULTANEOUS HEAT AND MULTI-COMPONENT MASS TRANSFER LIMITATIONS—I

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Abstract—In Part I, new theoretical treatments of the growth of a stationary spherical second phase as governed by simultaneous heat and multi-component mass transfer limitations are demonstrated. The new method is a direct extension of [1–3]. It demonstrates that the solution to these complex coupled cases can be related to the available uncoupled cases. Thus, treating the so-called "impurities" as components in the surrounding first phase, our results should include the growth of a spherical second phase as governed by simultaneous heat and mass transfer limitations in the presence of impurities as asymptotic cases.

NOMENCLATURE

$A(t)$,	function defined in equation (18), Part II.
$A_h(t)$,	$A(t)$ calculated from the heat transfer viewpoint;
$A_{mi}(t)$,	$A(t)$ calculated from the *i*th component mass transfer viewpoint;
$\dot{A}(t)$,	function defined in equation (18), part III:
$\tilde{A}_h(t)$,	$\tilde{A}(t)$ calculated from the heat transfer viewpoint:
$\tilde{A}_{mi}(t)$,	$\tilde{A}(t)$ calculated from the *i*th component mass transfer viewpoint:
$B(t)$,	function defined in equation (19), Part II;
$B_h(t)$,	$B(t)$ calculated from the heat transfer viewpoint;
$B_{mi}(t)$,	$B(t)$ calculated from the *i*th component mass transfer viewpoint:
$\tilde{B}(t)$,	function defined in equation (19), Part III:
$\tilde{B}_h(t)$,	$\tilde{B}(t)$ calculated from the heat transfer viewpoint;
$\tilde{B}_{mi}(t)$,	$\tilde{B}(t)$ calculated from the *i*th component mass transfer viewpoint:
B^g_h,	$= C_p(T_\infty - T_w)/L(T_w)$ [dimensionless];
B^g_{mi},	$= (C_{wi} - C_{\infty i})/(C_{di} - C_{wi})$ [dimensionless];
C_i,	mass fraction of *i*th solute [dimensionless];
C_{di},	second phase *i*th solute mass fraction [dimensionless];
$C_{\infty i}$,	*i*th solute mass fraction far away from the second phase sphere [dimensionless];
C_p,	specific heat of the surrounding first phase [cal/g °K];
$C_{\text{sat}\,1}(T, C_2, C_3, \ldots, C_{N-1})$,	first component (main solute) saturation mass fraction [dimensionless];
C_{wi},	*i*th solute mass fraction at the interface [dimensionless];
D_i,	effective Fick's diffusion coefficient of *i*th solute [cm²/s];
D/Dt,	material time derivative defined in text:
$f(t, \theta)$,	function defined in equation (42), Part II;
$\tilde{f}(t, \theta)$,	function defined in equation (41), Part III;
$G(t)$,	function defined in equation (17), Part II:

$G_h(t)$,	$G(t)$ calculated from the heat transfer viewpoint:
$G_{mi}(t)$,	$G(t)$ calculated from the ith component mass transfer viewpoint:
$\tilde{G}(t)$,	function defined in equation (17), Part III:
$\tilde{G}_h(t)$,	$\tilde{G}(t)$ calculated from the heat transfer viewpoint:
$\tilde{G}_{mi}(t)$,	$\tilde{G}(t)$ calculated from the ith component mass transfer viewpoint:
$H(t)$,	function defined in equation (41), Part II:
$H_h(t)$,	$H(t)$ calculated from the heat transfer viewpoint;
$H_{mi}(t)$,	$H(t)$ calculated from the ith component mass transfer viewpoint:
$\tilde{H}(t)$,	function defined in equation (40), Part III:
$\tilde{H}_h(t)$,	$\tilde{H}(t)$ calculated from the heat transfer viewpoint:
$\tilde{H}_{mi}(t)$,	$\tilde{H}(t)$ calculated from the ith component mass transfer viewpoint:
$L(T_w)$,	latent heat of phase transition (< 0 for endothermic: > 0 for exothermic) [cal/g];
N,	total number of solutes (or components) in the surrounding first phase [dimensionless]:
Na_h^g,	$\equiv (\rho/\rho_d . B_h^g \equiv (\rho/\rho_d) \times (C_p(T_\infty - T_w)/L(T_w))$ [dimensionless]:
Na_{mi}^g,	$\equiv (\rho/\rho_d) . B_{mi}^g \equiv (\rho/\rho_d) \times ((C_{wi} - C_{\infty i})/(C_{di} - C_{wi}))$ [dimensionless]:
p,	dummy integration variable:
$R, R(t)$,	instantaneous radius of the growing second phase sphere [cm]:
$R_h, R_h(t)$,	$R, R(t)$ calculated from the heat transfer viewpoint [cm];
$R_{mi}, R_{mi}(t)$,	$R, R(t)$ calculated from the ith component mass transfer viewpoint [cm]:
$\dot{R}, \dot{R}(t)$,	$\equiv dR/dt \equiv dR(t)/dt$ [cm/s];
R_0,	initial radius of the growing second phase sphere [cm]:
r,	radial coordinate reckoned from the center of the second phase sphere [cm];
s,	dummy integration variable:
t,	time (reckoned from the commencement of the growth process) [s]:
$T, T(r, \theta, t)$,	temperature [°K]:
T_w,	second phase sphere surface temperature [°K]:
T_∞,	temperature far away from the second phase sphere [°K]:
U_∞,	translational velocity of the center of the second phase sphere relative to the surrounding fluid at infinity [cm/s];
v_{rg},	radially spherically symmetric convective velocity field induced by mass transfer process itself [cm/s]:
v_{rt},	radial velocity component field induced by the translatory motion of the second phase sphere [cm/s]:
$v_{\theta t}$,	tangential velocity component field induced by the translatory motion of the second phase sphere [cm/s]:
x,	dummy integration variable:
y,	distance from the interface [cm].

Greek symbols

α,	$\equiv \lambda/(\rho C_p)$, thermal diffusivity of the surrounding first phase [cm^2/s]:
β,	growth constant [dimensionless]:
β_h,	β calculated from the heat transfer viewpoint [dimensionless]:
β_{mi}	β calculated from the ith component mass transfer viewpoint [dimensionless]:
ρ,	density of the surrounding first phase [g/cm^3]:
ρ_d,	density of the second phase [g/cm^3];
λ,	effective thermoconductivity of

	the surrounding first phase [cal/sm°K]:
μ,	viscosity of the surrounding first phase [cp]:
μ_d,	viscosity of the second phase [cp]:
κ	$= \mu_d/\mu$, ratio of viscosities [dimensionless]:
θ,	angle [radians];
τ,	dummy integration variable:
ξ,	dummy integration variable:
$\gamma(t)$,	function defined in equation (43), Part II:
$\gamma_h(t)$,	$\gamma(t)$ calculated from the heat transfer viewpoint:
$\gamma_{mi}(t)$,	$\gamma(t)$ calculated from the ith component mass transfer viewpoint
$\tilde{\gamma}(t)$,	function defined in equation (42), Part III:
$\tilde{\gamma}_h(t)$,	$\tilde{\gamma}(t)$ calculated from the heat transfer viewpoint:
$\tilde{\gamma}_{mi}(t)$,	$\tilde{\gamma}(t)$ calculated from the ith component mass transfer viewpoint:
$\phi(1 - \rho_d/\rho, \beta)$,	function defined in equation (11), Part I.

Subscripts

d,	pertaining to the second phase:
di,	pertaining to the second phase and the ith component:
h,	pertaining to the heat transfer viewpoint:
i,	pertaining to the ith component: if not specified, i runs from 1 to $N - 1$;
mi,	pertaining to the ith component mass transfer viewpoint:
sat 1,	first component saturated (pertaining to the equilibrium at the phase interface);
w,	at the interface, $r = R(t)$:
wi,	ith component at the interface:
0,	evaluated at $t = 0$:
∞i,	ith component far away from the center of the second phase sphere:
∞,	far away from the center of the second phase sphere.

Superscripts

g,	pertaining to multi-component second phase:
$\sim$,	pertaining to the fast moving case.

INTRODUCTION

GROWTH laws of stationary, slowly moving, and fast moving spherical second phase as governed by simultaneous heat and mass transfer limitations have been obtained recently [1–3]. The second phase can be a bubble (gas), a droplet (liquid), or a particle (solid). In the treatments of [1–3], the surrounding first phase is composed of solute and solvent, i.e. two-component environment. Now, we extend the treatment to the case when the surrounding first phase is composed of $(N - 1)$ solutes and solvent, i.e. N-component environment. Thereby, treating the so-called "impurities" as components in the surrounding first phase, our results should include the growth of a spherical second phase as governed by simultaneous heat and mass transfer limitations in the presence of impurities as asymptotic cases, as will be demonstrated later. In what follows, we will always take the first component as the main solute and Nth component as the solvent. In Part I, we treat the case when there exists no translatory motion of the center of the sphere, i.e. the spherical second phase is stationary with respect to the laboratory coordinates. In Part II, we treat the case when the spherical second phase is slowly moving. In Part III, we treat the case when the spherical second phase is fast moving. All the main results obtained in [1–3] when the surrounding first phase is two-component environment will be recovered as the asymptotic cases, as will be demonstrated in due course.

STATEMENT OF THE PROBLEM

The problem under consideration is as follows: A spherical second phase of negligible size, i.e. $R(0) = R_0 \approx 0$, is produced in an environment, i.e. the surrounding first phase, at time $t \mp 0$. The second phase can be a bubble (gas), a droplet (liquid), or a particle (solid). At time $t = 0$, the entire second phase is assumed to have attained a certain proper equilibrium temperature T_w, i.e. the wet bulb temperature, and remain at this temperature throughout the growth process. That is, one assumes that throughout the entire transient growth process a constant T_w exists, corresponding to a set of constant surface concentrations, $C_{\mathrm{sat}\,1}(T_w, C_{w2}, C_{w3}, \ldots, C_{wN-1})$ and C_{wi} $(i = 2, 3, \ldots, N-1)$, which must be found as part of the problem solution (see Discussion). At times $t > 0$, the spherical second phase grows in the surrounding first phase due to both heat and N-component mass transfer driving forces.

The stationary spherical second phase is characterized by the following parameters: density, ρ_d, latent heat of phase transition, $L(T_w)$ (< 0 for endothermic; > 0 for exothermic), and first component saturation concentration, $C_{\mathrm{sat}\,1}(T, C_2, C_3, \ldots, C_{N-1})$; the surrounding first phase is characterized by the following parameters: density, ρ, specific heat, C_p, effective thermoconductivity, λ, and effective Fick's diffusion coefficients, D_i $(i = 1, 2, \ldots, N-1)$. The first phase is initially at a uniform temperature T_∞, and concentrations $C_{\infty i}$ $(i = 1, 2, \ldots, N-1)$, while the second phase is assumed to have a uniform temperature T_w and concentrations C_{di} $(i = 1, 2, \ldots, N-1)$ throughout the growth process. Thus, the mass transfer process within the second phase is not considered here.

During the growth process, i.e. $t \geqslant 0$, the system is described by the following equations,

$$\frac{\mathrm{D}T}{\mathrm{D}t} = \alpha \nabla^2 T, \qquad R(t) \leqslant r \leqslant \infty \tag{1a}$$

$$\frac{\mathrm{D}C_i}{\mathrm{D}t} = \mathrm{D}_i \nabla^2 C_i, \qquad R(t) \leqslant r \leqslant \infty \tag{1b}$$

with

$$\frac{\mathrm{D}}{\mathrm{D}t} \equiv \frac{\partial}{\partial t} + \frac{R^2}{r^2} \cdot \left(1 - \frac{\rho_d}{\rho}\right) \cdot \dot{R} \cdot \frac{\partial}{\partial r}$$

and

$$\nabla^2 \equiv \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right)$$

$$T(r, 0) = T_\infty \tag{2a}$$

$$C_i(r, 0) = C_{\infty i} \tag{2b}$$

$$T(\infty, t) = T_\infty \tag{3a}$$

$$C_i(\infty, t) = C_{\infty i} \tag{3b}$$

$$T(R(t), t) = T_w \tag{4a}$$

$$C_i(R(t), t) = C_{wi} \tag{4b}$$

$$\rho_d \dot{R} = \frac{\lambda}{-L(T_w)} \cdot \left(\frac{\partial T}{\partial r} \right)_{r = R(t)} \tag{5a}$$

$$\rho_d \dot{R} = \frac{D_i \rho}{C_{di} - C_{wi}} \cdot \left(\frac{\partial C_i}{\partial r} \right)_{r = R(t)} \tag{5b}$$

$$R(0) = R_0 \approx 0 \tag{6}$$

where i runs from 1 to $N - 1$. $\alpha \equiv \lambda/(\rho C_p)$ is the thermal diffusivity of the surrounding first phase, and the first component surface concentration is assumed to be $C_{w1} = C_{\mathrm{sat}\,1}(T_w, C_{w2}, C_{w3}, \ldots, C_{wN-1})$. The problem is to find the *a priori* unknown interface temperature T_w and concentrations C_{wi} $(i = 1, 2, \ldots, N-1)$ and obtain the growth law of the second phase, $R(t)$.

METHOD OF SOLUTION

The key to this physically important problem is to recognize that the growth laws obtained from either heat or N-component mass transfer viewpoints must be identical. Thus, one obtains the compatibility conditions from which T_w and C_{wi} $(i = 1, 2, \ldots, N-1)$ are calculated (see below).

From the heat transfer viewpoint, i.e. the set of equations (1a), (2a), (3a), (4a), (5a) and (6), the temperature variable $T(r, t)$ satisfies the same boundary-value problem as in [4–6]. Thus, one gets [4–6]

$$\boxed{\frac{T-T_\infty}{T_w-T_\infty}=\frac{\int_{[r/2\sqrt{(\alpha t)}]}^{\infty} x^{-2}\exp\left[-x^2-2.(1-\rho_d/\rho).\beta_h^3x^{-1}\right]\mathrm{d}x}{\int_{\beta_h}^{\infty} x^{-2}\exp\left[-x^2-2.(1-\rho_d/\rho).\beta_h^3.x^{-1}\right]\mathrm{d}x}} \quad (7)$$

where

$$R_h = 2\beta_h.\sqrt{(\alpha t)} \quad (8)$$

and β_h is given by

$$Na_h^e = -\phi\left(1-\frac{\rho_d}{\rho},\beta_h\right) \quad (9)$$

with

$$Na_h^e \equiv \frac{\rho}{\rho_d}.B_h^e \equiv \frac{\rho}{\rho_d}.\frac{C_p(T_\infty-T_w)}{L(T_w)} \quad (10)$$

and the function $\phi(1-\rho_d/\rho,\beta)$ defined by

$$\phi\left(1-\frac{\rho_d}{\rho},\beta\right) \equiv 2\beta^3\exp\left[\beta^2+2\times\left(1-\frac{\rho_d}{\rho}\right).\beta^2\right].\int_{\beta}^{\infty}x^{-2}\exp\left[-x^2-2\times\left(1-\frac{\rho_d}{\rho}\right).\beta^3.x^{-1}\right]\mathrm{d}x. \quad (11)$$

From the ith component mass transfer viewpoint, i.e. the set of equations (1b), (2b), (3b), (4b), (5b) and (6), the concentration variable $C_i(r,t)$ satisfies the same boundary-value problem as in [4–6]. Thus, one gets [4–6]

$$\boxed{\frac{C_i-C_{\infty i}}{C_{wi}-C_{\infty i}}=\frac{\int_{(r/2\sqrt{D_i t})}^{\infty} x^{-2}\exp\left\{-x^2-2.[1-(\rho_d/\rho)].\beta_{mi}^3x^{-1}\right\}\mathrm{d}x}{\int_{\beta_{mi}}^{\infty} x^{-2}\exp\left\{-x^2-2.[1-(\rho_d/\rho)].\beta_{mi}^3x^{-1}\right\}\mathrm{d}x}} \quad (12)$$

where

$$R_{mi}(t) = 2\beta_{mi}\sqrt{(D_i t)} \quad (13)$$

and β_{mi} is given by

$$Na_{mi}^e = -\phi\left(1-\frac{\rho_d}{\rho},\beta_{mi}\right) \quad (14)$$

with

$$Na_{mi}^e \equiv \frac{\rho}{\rho_d}.B_{mi}^e \equiv \frac{\rho}{\rho_d}.\frac{C_{wi}-C_{\infty i}}{C_{di}-C_{wi}} \quad (15)$$

and the function $\phi(1-\rho_d/\rho,\beta)$ is given in equation (11). The uniqueness of the growth law of the second phase, i.e. $R_h(t) = R_{m1}(t) = R_{m2}(t) = \ldots = R_{mN-1}(t) = R(t)$, gives the following compatibility conditions

$$\begin{aligned}\beta_h.\sqrt{\alpha} &= \beta_{m1}.\sqrt{D_1}\\ &= \beta_{m2}.\sqrt{D_2}\\ &= — \quad — \quad —\\ &= \beta_{mN-1}.\sqrt{D_{N-1}}\end{aligned} \quad (16)$$

where β_h and β_{mi} are given by equations (9) and (14), respectively. Owing to the $C_{\text{sat}1}(T, C_2, C_3, \ldots, C_{N-1})$ relation, equation (16) determines a unique value of T_w and a unique set of values for C_{wi} ($i = 1, 2, \ldots, N-1$). Then, the required growth law of the second phase follows either from equations (8)–(11) or equations (13)–(15) and (11). The comprehensive plot of equations (9) or (14) was already shown in [1], which is equivalent to Scriven's Fig. 1 [6].

In order to find the values of T_w and C_{wi} ($i = 1, 2, \ldots, N-1$), one has to use a trial-and-error method. However, in certain physically important asymptotic cases, T_w and C_{wi} ($i = 1, 2,$

$\ldots, N-1$) can be readily obtained analytically (see [1] for the case of $N = 2$). In particular, in case $D_1 = D_2 = \ldots = D_{N-1} = \alpha$, which is reasonable for a gaseous first phase, the compatibility conditions, equation (16), are exactly satisfied for any ρ_d/ρ if one merely sets $B_h^g = B_{m1}^g = B_{m2}^g = B_{m3}^g = \ldots = B_{mN-1}^g$, i.e.

$$\frac{C_p \cdot (T_\infty - T_w)^{(1)}}{L(T_w)} = \frac{C_{w1} - C_{\infty 1}}{C_{d1} - C_{w1}} = \frac{C_{w2} - C_{\infty 2}}{C_{d2} - C_{w2}} = — — — = \frac{C_{wN-1} - C_{\infty N-1}}{C_{dN-1} - C_{\infty N-1}} \tag{17}$$

where $C_{w1} = C_{\mathrm{sat}\,1}(T_w, C_{w2}, C_{w3}, \ldots, C_{wN-1})$. Owing to the $C_{\mathrm{sat}\,1}(T, C_2, C_3, \ldots, C_{N-1})$ relation, equation (17) determines a unique value of T_w and a unique set of values for C_{wi} ($i = 1, 2, \ldots, N-1$). Then, the required growth law of the second phase follows either from equations (8)–(11) or (13)–(15) and (11). To fix the ideas, we will consider the following physically important asymptotic cases.

Case 1. $N = 2$ (18)

When $N = 2$, i.e. two-component environment, the main results obtained in [1] are recovered, as expected.

Case 2: $N = 3$ (19)

When $N = 3$, i.e. three-component environment, equation (16) degenerates into the following form

$$\beta_h \cdot \sqrt{\alpha} = \beta_{m1} \cdot \sqrt{D_1} = \beta_{m2} \cdot \sqrt{D_2} \tag{20}$$

where β_h and β_{mi} ($i = 1, 2$) are given by equations (9) and (14), respectively. Treating the first component as the main solute, the second component as the impurity, and the third component as the solvent, this is the case when the growth of the spherical second phase is governed by simultaneous heat and mass transfer limitations in the presence of an impurity. Owing to $C_{\mathrm{sat}}(T, C_2)$ relation, equation (20) determines unique values for T_w, $C_{w1} = C_{\mathrm{sat}\,1}(T_w, C_{w2})$, *and* C_{w2}. Then, the required growth law of the second phase follows either from equations (8)–(11) or (13)–(15) and (11).

DISCUSSION

Before we proceed further, we feel it is necessary to recapture the fundamental assumptions used in Part I. First of all, it is assumed that solute and heat diffusions in the surrounding first phase are adequately described by unsteady state diffusion equations with effectively constant Fick's diffusion coefficients and an effectively constant thermoconductivity. It is assumed that all the parameters characterizing second and first phases are effectively constant and there exists a local equilibrium relationship, $C_{\mathrm{sat}\,1}(T_w, C_{w2}, C_{w3}, \ldots, C_{wN-1})$ at $r = R(t)$ throughout the growth process. The assumption of negligible initial size has been discussed in [1] and shown to be valid. The compatibility conditions, equation (16) are the necessary and sufficient conditions for the existence of the stated constant interface conditions solution, i.e. it guarantees the uniqueness of the growth law of the second phase, $R(t)$. Thus, the basic assumption of strictly constant T_w and C_{wi} ($i = 1, 2, \ldots, N-1$) is automatically justified *a posteriori* for the second phase problems of the type considered here. Physically, the necessary and sufficient compatibility conditions mean that the second phase can grow if one maintains $T(\infty, t) = T_\infty$ *and* $C_i(\infty, t) = C_{\infty i}$ ($i = 1, 2, \ldots, N-1$) throughout the growth process.

CONCLUSIONS

In Part I, an exact treatment of the growth of a stationary spherical second phase in the presence of simultaneous heat and multi-component mass transfer limitations has been demonstrated. In general, a trial-and-error method must first be used to solve the compatibility conditions, equation (16), to obtain the *a priori* unknown second phase temperature and surface concentrations. Having thus determined T_w and C_{wi} $(i = 1, 2, \ldots, N-1)$, the growth law of the second phase is then readily obtained. Treating the so-called "impurities" as components in the surrounding first phase, our results should include the growth of a spherical second phase as governed by simultaneous heat and mass transfer limitations in the presence of impurities as asymptotic cases.

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LOI DE CROISSANCE D'UNE SECONDE PHASE SPHERIQUE GOUVERNEE PAR DES CONDITIONS DE TRANSFERTS SIMULTANES DE CHALEUR ET DE MASSE—I

Résumé—On considère dans une première partie, de nouveaux traitements théoriques de la croissance d'une seconde phase sphérique et stationnaire, pilotée par des limitations de transferts simultanés de chaleur et de masse à plusieurs composants. La nouvelle méthode est une extension directe de celle des références (1–3). Il est démontré que la solution de ces cas couplés compliqués peut être reliée à des cas non couplés connus. Ainsi, traitant les "impuretés" comme des composants dans la première phase environnante, les résultats peuvent inclure, comme des cas asymptotiques, la croissance d'une seconde phase sphérique gouvernée par des limitations de transferts simultanés de chaleur et de masse en présence des impuretés.

WACHSTUMGSGESETZ EINER KUGELFÖRMIGEN SEKUNDÄRPHASE FÜR GLEICHZEITIGEN WÄRME- UND VIEL-KOMPONENTEN-STOFFÜBERGANG—I

Zusammenfassung—Im Teil I werden neue theoretische Betrachtungen über das Wachstum einer durch gleichzeitigen Wärmeübergang und Mehrkomponentenstoffübergang begrenzten stationären kugelförmigen Sekundärphase angestellt. Die neue Methode ist eine unmittelbare Erweiterung von [1–3]. Sie zeigt, dass die Lösung dieser komplex überlagerten Fälle auf vorhandene ungekoppelte Fälle übertragen werden kann. Indem man diese sogenannten "Unreinheiten" als Komponenten in der umgebenden ersten Phase behandelt, sollten unsere Beziehungen das Wachstum einer durch gleichzeitigen Wärme- und Stoffübergang begrenzten kugelförmigen Sekundärphase in Anwesenheit der Unreinheiten als asymptotische Fälle enthalten.

ЗАКОН РОСТА СФЕРИЧЕСКОЙ ВТОРОЙ ФАЗЫ ПРИ ОДНОВРЕМЕННОМ ПЕРЕНОСЕ ТЕПЛА И МНОГОКОМПОНЕНТНОЙ МАССЫ—I

Аннотация—В части I показана новая теоретическая трактовка роста стационарной сферической второй фазы при совместном переносе тепла и многокомпонентной массы.

Новый метод является прямым продолжением метода, изложенного в [1–3]. Он показывает, что для решения таких сложных задач взаимосвязанного тепло-и массообмена можно использовать известные для простых задач решения. Таким образом, рассматривая так называемые «примеси» в качестве компонент окружающей их первой фазы, необходимо включать как асимптотический случай рост сферической второй фазы, определяемый закономерностями одновременного тепло-и массопереноса при наличии примесей.